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Flame Propagation Through Swirling Eddys, A Recursive Pattern

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Abstract – Computed flame motion through and between swirling eddys exhibits a maximum advancement rate which is related to the time duration of flame motion between eddys. This eddy spatial structure effect upon the apparent turbulent flame speed appears to be similar to the square-root dependence observed in wrinkled flamelet data. The rate-limiting behavior at one eddy length-scale can be removed by inclusion of smaller eddys which reside between the larger eddys. This large-eddy, small-eddy concept yields a recursion relation and repeated functional iteration can be done to approximate a desired flame speed relation. As an example, an iteration to produce $S_T \ln S_T = u'$ *is given for the range of* u' *observed in liquid flames. Currently, the iteration process is a post-diction of flame speed, but if a universality can be developed, then a predictive theory of turbulent flame propagation might be achieved.*

Prediction of Premixed Turbulent Flame Speed

Determination of the propagation rate of a flame through a turbulent gas with premixed fuel and air is a very practical problem with many attempts since the work by Damköhler in 1940. He suggested that if the turbulent motion did not change the local flame speed from its laminar value S_L, then the effective turbulent flame speed was proportional to the total flame area divided by the cross-sectional flow area: $S_T/S_L = A_T/A$. But, determination of the total flame surface area by direct measurement or calculation has not been possible. A year after Damköhler, Kolmogorov proposed the inviscid energy cascade in turbulent motion and so, the idea that motions at different wavelengths have

a certain power relation has been applied to the flame propagation problem. Application of Kolmogorov's 1941 theory to the premixed flamelet regime leads to $S_T \sim u'$, where the energy dissipation rate is u'^3/L and u' is the rms velocity with L the velocity integral length scale (see Clavin & Siggia, 1991). In this model the turbulent propagation rate does not depend upon the laminar flame speed, nor is there an upper limit on S_T as u' increases. Also there is no explicit description of the spatial structure of the tubulent flow.

Peters introduced the corrugated flamelet regime with the Gibson length scale for conditions where the flame thickness is smaller than the smallest turbulent scale and the laminar flame velocity is small compared to the velocity fluctuations (1988). The Gibson length corresponds to the smallest wavelength of the flame wrinkles, because turbulent eddys which are smaller than L_G have rotational velocities smaller than S_L and so can not distort the flame surface. Kerstein (1988a) suggested that each active eddy length scale between L_G and L will burn out in the time required for one turn over time of that particular eddy, but the rate of propagation within that eddy is enhanced by the smaller eddys. If the fractal dimension is 7/3, then this consumption of an eddy in one turn over time will occur for all eddy sizes over the fractal range from L_G to L. The estimated turbulent flame brush thickness is proportional to L and the linear relation between S_T and u' is obtained.

Another approach by Kerstein (1988b) estimates the turbulent flame brush in two ways: 1) $\delta_T \sim \tau S_T$ and 2) $\delta_T \sim D/S_T$. Here τ is the consumption time for an eddy and the turbulent diffusivity D is replaced with $u'L$, giving

$$S_T^2 = u'L/\tau.$$

The consumption time is estimated from the growth of flame area by strain rate and Kerstein develops a simple derivation of Yakhot's result obtained via renormalization techniques: $S_T/S_L = \exp[(u'/S_T)^2]$. A different estimate for the consumption time is based on an eddy spacing proportional to the Taylor length scale λ (see, Gülder, 1990) and the laminar flame speed, yielding: $\tau \sim \lambda/S_L$. This latter choice leads to

$$\frac{S_T}{S_L} = \sqrt{\frac{u'}{S_L}\frac{L}{\lambda}}$$

and using the volume averaged energy dissiapation $\epsilon = 15\nu u'^2/\lambda^2 \sim u'^3/L$ we have

$$\frac{S_T}{S_L} = \sqrt{\frac{u'}{S_L}\frac{Re_\lambda}{15}} \tag{1}$$

where Re_λ is the Reynolds number based on the Taylor length scale. In a latter section, we give results that appear to agree with the square-root behavior given by Equation (1); these results are from numerical simulations of passive flame propagation within Navier-Stokes turbulence and within defined two-dimensional flows.

The above conceptual models of flame propagation through a turbulent flow leave out the effects of volume expansion created by the chemical reaction. However, for the wrinkled flamelet regime, Gülder shows that a sampling of the experimental data may follow the square-root power-law given above. A power-law with an exponent less than unity matches the character of the data which has a linear relation when $u' \sim S_L$, but changes to an apparent maximum for S_T/S_L when $u' >> S_L$. This changing relationship is also called a "bending" effect (see Peters, 1988 and Sivashinsky, 1988). Volume expansion, by damping the turbulence in the unburnt gas and thereby reducing the flame wrinkling, could be one cause of this bending effect. In the present paper we propose that the spatial structure of turbulence could be another cause of this bending effect. All the computer simulations on which our spectulation is based do not include heat release and so, we can not judge the relative importance of these two possibilities for the bending effect.

The vital observation of the turbulence simulations is that the intense vortical regions are tube-like in shape and that these tubes appear to be finite in length. Furthermore, the number density or the spacing of these tubes is such that they are not space filling, see Figure 1. The non-space filling feature is consistent with Kolmogorov's newer theories regarding the energy cascade which include the intermittency nature of turbulence (see Gibson, 1991). Current direct simulations of turbulence have been done with grid meshes of $\approx 100^3$, and so the observed turbulence structure will need confirmation by larger calculations. However, the current three-dimensional turbulent flame propagation simulations do show how the spatial structure of turbulence affects the turbulent flame speed.

Turbulence Structure Effect Upon Flame Propagation

The effect of spatial structure is most obvious in a two-dimensional flow composed of

vortical eddys separated by regions which are non-swirling by comparison, this yields a flame propagation which is composed of two types of advancement. One type is the flame propagation within the vortical eddy and the other type is the flame motion between eddys. Within an eddy, the swirling flow will be characterized as having rigid body rotation (zero shear) near the axis, a maximum swirl velocity at radius R_m and a near zero swirl velocity for radial locations beyond three or four times R_m, see Figure 2. When the swirl velocity U_m is greater than S_L then this eddy will wrinkle the flame and when U_m is much greater than S_L this eddy will form pockets of unburnt gas. From time-dependent, two-dimensional simulations of flamelet motion it becomes very obvious that when the flame contacts the eddy, contact in the sense that some part of the flame surface is near the maximum swirl radius, then the flame front begins to advance around the eddy circumference. The rate of advancement is $U_m + S_L$. After an elapsed time of approximately half of a turn over time, the flame front will now be located a distance of $2R_m$ from the flame location that existed before the contact. When $U_m >> S_L$ then the flame front has been significantly advanced during this fraction of the eddy turn over time. This is the eddy advancement of the flame. The non-eddy advancement corresponds to flame propagation in regions between eddys and the flame duration in the non-swirling regions becomes the rate-limiting step when the swirl velocity is much larger than the flame speed.

The fact that flow spatial structure may limit the maximum flame propagation speed will be illustrated by numerical simulations of passive flame motion. The two-dimensional eddy configuration to be presented is certainly a contrived flow pattern. However, it has been contrived to match a feature of turbulent flow, namely, the intense swirling regions are not space filling. The utility of these two-dimensional simulations is the ease with which one can determine a model of flame advancement which exhibits the rate-limiting behavior found in the nonlinear simulations. Examination of flamelet propagation within three-dimensional Navier-Stokes turbulence reveals the same rate-limiting behavior as found in the contrived flow. Thus, flows in which the most intense vortical motions are not space filling will exhibit a bending effect in the relation of S_T and u'.

The eddy structure shown in Figure 1 would be very simple if all the eddys had the same swirl speed, swirl radius and tube length. Only the arrangement in three-dimensional

space and the number density would need specification (assuming that the eddy lifetime is longer than the flame passage time). While not proven at this time, we do suggest that the simple model is reasonable. This concept was developed by trying to make the Burgers' vortex flow pattern fit the strain rate behavior observed in turbulence simulations. In order to do so, the axial strain rate of the Burger's vortex is related to the large-scale parameters u'/L and the vortex circulation is a fixed multiple of the kinematic viscosity. This makes the eddy radius comparable to the Taylor length scale. Confirmation of this Burgers' vortex model must await further simulations and the investigation of these simulations. The current value of these suggestions is that they reduce the number of free parameters in the structural description of turbulent flow and they show how the flow structure determines the turbulent flame speed – in the zero heat-release limit.

We first present the two-dimensional simulations which only include a single length scale of eddys. The functional form which describes these results also gives a good representation of the three-dimensional Navier-Stokes results. We then make the assumption that these eddy flow patterns could be repeated on larger length scales and so use the single-scale form as a recursion relation.

Two-Dimensional, Single-Scale Eddys

We treat a passive flame which has a propagation speed of S_L and is moving through a flow composed of swirling eddys. The swirling motion of each eddy is confined within a diameter D and the eddys are spaced at distance L and do not overlap, $L > D$. This collection of eddys creates a root-mean-square velocity u', and between the eddys the flow speed is zero in comparison with u', which we will refer to as the quiet zone. When $u' > S_L$ the flame advancement has two distinct paths: 1) propagation plus convection within an eddy, and 2) propagation across the quiet zone to reach the next eddy. The effective velocity for the flame advancement is

$$S_f = \frac{L}{t_1 + t_2} \tag{2}$$

where $t_1 = l_1/(S_L + u')$ and $t_2 = l_2/S_L$. The path length l_1 is proportional to the eddy circumference when $u' > S_L$ and reduces to the eddy diameter when $u' < S_L$. The path

l_2 is a representative distance of the quiet zones. The front speed is

$$S_f = \frac{L}{\frac{l_1}{S_L + u'} + \frac{l_2}{S_L}} \qquad = \frac{S_L + u'}{\frac{l_1}{L} + \frac{l_2}{L}(1 + u'/S_L)} \tag{3}$$

or, with $S_L = 1$, in the form of

$$S_f = \frac{(1 + u')}{(a + bu')} \tag{4}$$

with $a > 1$ and $b < 1$ for eddys that almost fill the available space. When u' increases without changing the number of eddys or the ratio D/L, then the maximum front speed becomes $1/b = L/l_2$. Thus, the quiet zones limit the flame advancement and cause a *bending* effect in the relation S_f vs u'.

An illustration of this propagation is shown in Figure 3 in which two eddys have been placed in a channel and the propagation dynamics have been solved with the G equation formulation

$$\frac{\partial G}{\partial t} + \mathbf{u} \cdot \nabla G = S_L |\nabla G| \tag{5}$$

(Kerstein *et al.,* 1988; Ashurst *et al.,* 1988). Letting the total flame surface equal the front advancement rate in this stationary system ($< |\nabla G| >= S_f$), then a fit of $< |\nabla G| >$ using Eq. (4) yields $a \approx 1.05$ and $b \approx 0.3$, see Figure 4.

There is an apparent departure of the calculated response from the behavior predicted by Eq. (4) when $u' \approx 2.5 S_L$. This departure could be the onset of unburned pockets within the eddy core: the rotation rate has convected the flame clear around the eddy and further increases in rotation rate can not further increase the flame area *and* with this complete wrap around there is the formation of islands of unburnt. Even with heat release, large eddy rotation can create islands (see the two-dimensional simulations of Ashurst & McMurtry, 1989). Flame pinching leading to pockets has been studied by Joulin & Sivashinsky (1991) in a non-swirling flow. Pocket formation and its transient effect could be included as a correction to Eq. (4) when $u' > 2S_L$. The change in flame curvature seen in Figure 3 for $u' < S_L$ and for $u' > S_L$ indicates that the path length l_2 in Eq. (2,3) might have a change at $u' > 2S_L$. From Figure 4, it appears that this path change should increase the value of the b coefficient and hence the maximum flame speed is reduced. A change in l_2 could

be a change in the time to reach the maximum swirl radius as the flame shape acquires a very large curvature.

Another two-dimensional eddy flow with the rate-limiting behavior is the square-eddy flow given in Figure 5 of Ashurst & Sivashinsky (1991): $u = -\sqrt{2}u' \cos kx \sin ky; v = \sqrt{2}u' \sin kx \cos ky$. The maximum swirl velocity is at the edge of the eddy in this flow pattern, and so the eddys are not separated by quiet zones. However, a fit of the computed flame speeds yields results similar to previous two-dimensional eddy results: the coefficients for the square-eddys are $a = 1.07$ and $b = 0.11$. The smaller b value in the square-eddy flow gives a larger maximum flame speed than is possible for the same spatial average of u' in the round-eddy configuration given above. In the square-eddy flow the quiet zone effect is created by the stagnation region between four eddys (the flame points are in the stagnation regions in sequence 4 of Figure 5 cited above). So, we may conclude that two-dimensional swirling motions will inherently have quiet regions which limit the maximum flame speed. The quiet-zone effect on flame propagation may also occur in three-dimensional turbulence due to the non-space filling nature of the turbulence structure.

Three-Dimensional Flame Propagation

Three-dimensional Navier-Stokes turbulence simulations have been combined with the G equation to determine the flame speed dependence upon u'/S_L. In these simulations the turbulent kinetic energy is maintained at a constant value by weak forcing of the large-scale strain rate. The flame curvature distribution given by these simulations, at $S_L = 2u'$, has good agreement with experimentally determined curvature, see Shepherd & Ashurst (1992). The flame speed results given in Figure 5 were obtained in a periodic cube with 32^3 grid-cells, the value of u' is unity and the kinematic viscosity is $\nu = u'L/500$ where L is the cube edge length, also equal to unity. The measured Taylor length scale is $\lambda \sim L/8$, resulting in $Re_\lambda \sim 60$. The laminar flame speed S_L was varied while the values of u' and ν were fixed. Each turbulent flame speed value is a time average over the same turbulence simulation, that is each initialization of the G equation has the same flow solution, the averaging time-period is $32L/u'$, about 28,000 time-steps. A fit of the three-dimensional,

Navier-Stokes flame speed results using Eq. (4) yields $a = 0.9021$ and $b = 0.1585$, values which are similar to the two-dimensional eddy results.

Examination of the computed turbulence reveals that the intense vortical eddys are separated by quiet zones and so, flame speed behavior in swirling flows may not depend upon flow dimensionally. Details of the turbulence structure are given by She *et al.* (1990). A noteworthy feature for modeling flame propagation is that a random flow, but with the Navier-Stokes energy spectrum, does not have the same vortical structure as the turbulent solution (see pictures in the *Nature* article).

An analysis of preliminary numerical flame speed results indicated that a square-root behavior might be the best fit, see Wirth & Peters (1992). Therefore, in Figure 6, the current 32^3 results are plotted versus $\sqrt{u'/S_L}$. The slope of the straight line is 1.32. We have used this square-root behavior in order to determine the a, b coefficients in a simple manner by expressing Eq. (4) in terms of $\sqrt{u'/S_L}$. Let $x^2 = u'$ and then Eq. (4) is

$$S_f(x) = \frac{1 + x^2}{a + bx^2} \tag{6}$$

and the slope of this function in terms of x is $2x(a - b)/(a + bx^2)^2$. For $u' = S_L$, denote the slope as m_1 and the flame speed value as $S_f(1)$ then

$$(a - b) = 2m_1/S_f^2(1) \qquad (a + b) = 2/S_f(1). \tag{7}$$

This procedure has been used to determine the coefficients of the two-dimensional eddy flows given previously. Examination of the dependence of the a, b coefficients, given by Eq. (7), upon the slope m_1 and the flame speed $S_f(1)$ shows that the slope m_1 has a stronger influence on b than on a, and the opposite behavior occurs for $S_f(1)$.

The square-root behavior seen in Figure 6 is consistent with the turbulence model given in Eq. (1), except for small values of u' where the numerical results do not appear to converge towards the value of $S_T = S_L$. In application of the model given in Eq. (1), Gülder replaces the left-hand side with $(S_T/S_L) - 1$ so that as $u' \to 0$, then $S_T \to S_L$. A recent analysis of weak, turbulent flame propagation indicates that neither the square-root nor the quadratic behavior will occur in a random field. Instead, a 4/3 power-law is realized in random fields when $u' << S_L$ (Kerstein & Ashurst, 1992). Therefore, the

dashed line connecting to the laminar flame speed value in Figure 6 is obtained by passing a 4/3 power relation through the first data point. The resulting coefficient is of order unity (1.43) and the smoothness of the transition is agreeable.

Flamelet Reynolds Number Dependence

Gülder (1990) has shown that experimental data for the wrinkled flamelet regime does appear to follow the square-root behavior given in Eq. (1). We have used the Taylor based Reynolds number as that quantity can be directly calculated in the simulations. The computational domain is not large enough to obtain a reliable measure of the integral scale, and the grid size is too coarse for the Kolmogorov scale, however the current simulations do have good agreement with strain-rate distributions obtained in larger-grid simulations. Therefore, we determine the Taylor Reynolds number and use $\sqrt{Re_L} = Re_\lambda = Re_\eta^2$ to relate the different scales. From Eq. (1), the estimated slope for the 32^3 results would be $\sqrt{60/15} = 2$ whereas the numerical simulations yield 1.32. A few simulations have been done with a 64^3 grid and, with this better resolution, the Taylor length scale changes to $\sim L/9$, giving an estimated slope, from Eq. (1), as $\sqrt{55/15} = 1.91$. Connecting the first and second points obtained with 64^3 yields a slope of 1.91. The uncertainity in the numerical values is at least $\pm 5\%$. Even larger grid simulations have not been done, but with the latter comparison, it appears that the consumption time estimate based on the Taylor length scale and the laminar flame speed yield a turbulent flame brush model which agrees with the numerical results.

Simulations with other viscosity values would allow comparison with the Reynolds number scaling. The current conditions are at the border of the flamelet regime given by Gülder, the bounds being $\eta > 1.5(\nu/S_L)$ and $Re_L < 3200$ or $Re_\lambda < 57$ (the reason for the Reynolds limit is not given, nor is it consistent with other flame diagrams, *cf.* Peters, 1988). Anyway, from Gülder's view the current simulations are at the upper range of Reynolds number. The curent simulations may also be the minimum Re_λ for turbulent energy dissipation to be independent of Reynolds number. For example, in grid turbulence, Sreenivasan (1984) shows that the turbulent energy dissipation, normalized by u'^3/L, depends upon Reynolds number when $Re_\lambda < 50$. This dissipation Reynolds dependence could be evidence that the vortical structure is different in this regime, and

so flame propagation could also have a different Reynolds number behavior. Experimental determination of turbulent flame speed could also have a dependence upon how the turbulence is generated. Sreenivasan presents results in which changes in grid geometry, from square holes to parallel rods, yields factors of two in the downstream dissipation value. Such grid effects might explain the larger coeficient in the flame results of Liu & Lenze cited by Gülder.

Multiple-Scale Eddys

So far we have considered only a single scale of eddys with quiet zones between eddys. The three-dimensional Navier-Stokes simulations are essentially a single-scale vortical structure, probably due to the small grid size. In the spirit that nature repeats patterns at different length scales, we conjecture what would be the effect upon the flame speed if there were a hierarchy of eddy scales. However, we wish to think of the smaller eddys as occurring between the larger eddys, and not within them. We exclude the small eddys from the region occupied by the larger eddys because the differential rotation caused by the intense vorticity removes velocity fluctuations in the large swirl regions. At larger radial distances from an eddy, the shearing motion becomes the negative of the swirling motion and irrotational exists. In this outer region of the large eddy, a small eddy would be convected but not distorted. Therefore, if we let the in-between zones of large eddys be occupied by similar eddys on a smaller length scale, then the time t_2 in Eq. (2) should be changed to

$$t_2 = l_2 / S_{f_2} \tag{8}$$

where S_{f_2} replaces S_L in what was previously assumed to be a quiet zone and so Eq. (3) changes to

$$S_{f_1} = \frac{S_L + u'}{\frac{l_1}{L} + \frac{l_2}{L}(S_L + u')/S_{f_2}}. \tag{9}$$

It appears that by using functional iteration we may include the effects of eddys at smaller, or larger, length scales. We change Eq. (4) to

$$S_{f_n} = \frac{1 + u'}{(a - b) + b(1 + u')/S_{f_{n-1}}} \tag{10}$$

where S_{f_n} is the effective flame speed for eddys at spacing $l_n(> l_{n-1})$ and at the smallest possible eddy length scale l_1, S_{f_0} is $S_L(= 1)$ and we have Eq. (4). A flow with p levels of eddys will have a maximum speed of S_L/b^p and so the bending effect will still appear, but at a larger value of u' than a flow with $p-1$ levels. In this eddy model the propagation within an eddy is at $S_L + u'$ and only between the eddys is there an enhanced propagation of $S_{f_{n-1}}$, which is different than Sivashinsky's model (1988) where each length scale l_n distorts a flame that travels with an apparent speed of $S_{f_{n-1}}$. The eddy pattern proposed here is that of smaller eddys *between* larger eddys rather than eddys *within* larger eddys.

Using this functional iteration procedure we can take the numerical simulation as the smallest eddy size and expand the results to a larger volume with eddys active on many length scales. An example of this procedure is given in the next section.

Comparison with Liquid Flames

Ronney and co-workers (Shy *et al.*, 1992) have used a chemically reacting, propagating front in a liquid to simulate thin premixed flames which have many of the popular theoretical assumptions: density, S_L and transport coefficients are all constant. They obtain flame speed versus turbulence intensity in two different configurations: 1) Taylor-Couette and 2) capillary-wave flow. In these liquid flames they can reach values of $u' = 1000S_L$ and more. Over a wide range of u' they find the Taylor-Couette results are reasonably described by $S_T = \exp(u'/S_T)$, with $S_L = 1$.

We use this observation of their experimental results to determine values in the functional iteration procedure. Using Eq. (10) in a repeated fashion, we assume that the introduction of the next level of eddys should produce a flame speed S_{f_n} which does not exceed the value given by the relation $S_T \ln S_T = u'$.

In Figure 7, the Navier-Stokes a, b values from above are maintained constant as new levels of eddys are introduced. In order to achieve no overshoot, as described above, we find that the turbulence parameter u' depends on the eddy level, in fact decreasing as we include larger length scales. Rather than kinetic energy of the flow, it is closer to the eddy flow pattern to think of u' as representating the eddy vorticity times the swirl radius

– perhaps the Burgon structure (Ashurst, 1992). In a Burgers' vortex the vorticity has a Gaussian radial dependence and the turnover time at the maximum swirl radius r_m is $t_u \approx 1/s$ where s is the axial strain rate (constant s gives a steady, viscous flow). The maximum-swirl radius scales as $r_m \propto \sqrt{4\nu/s}$ and with $u' \propto r_m/t_u$ then $u' \propto \sqrt{4\nu s}$. Hence, the parameter u' should increase as the length scale decreases because smaller scales have larger strain rates.

The introduction of one eddy level larger than the Navier-Stokes simulation, Figure 7a, with $u'_2/u'_1 = 0.42$, provides agreement for $u' < 30$. In Figure 7b, with $u'_3/u'_2 = 0.62$, the agreement goes beyond 200 in u'. In Figure 7c, with $u'_4/u'_3 = 0.70$, there is agreement up to $u' = 1000$. While these assumed ratios of swirl intensity are not to be taken too seriously, it is worth noting that only a few eddy levels are needed to give the appearence of a turbulent flame speed that is similar to experimental expressions. *If* this eddy structure is close to the actual physical motion, then a turbulent flow with only two or three eddy levels would appear to have $S_T \sim u'$ when $u' > S_L$.

Features of the Eddy Model

This flame propagation model depends on the fluid vorticity structure, the eddys. From experiments and turbulence simulations, we know that two eddys can not be too close to each other if they are to remain as distinct eddys. Between a pair of eddys there usually is a local stagnate region, and it is this quiet zone around the eddy which is the rate limiting step in flame propagation when the eddy swirl speed becomes larger than the flame speed. However, if it is the nature of turbulence to create similar flow patterns at different length scales, then in the quiet zone there can be another pair of eddys, but much smaller in size. These small-scale eddys enhance the quiet zone flame propagation and so the rate limiting step of the larger scale has been changed by the physical location of the small scales. Note that this model has the smaller eddys between the larger ones, not within them. This eddy pattern can be repeated down to the level where the molecular structure causes departure from the continuum solution or to the level where the energy flux is not strong enough to create distinct eddys. The Burgers' vortex is a good candidate for the

eddy structure, this viscous, dissipative flow has a steady solution when the axial strain rate s is constant. The vortex vorticity times swirl radius may scale as $\sqrt{\nu s}$, which we assume is the relevant parameter u' in the flame speed relation. By adjusting u'_n for each level of eddys at length l_n, we have shown that it is possible to approximate a flame speed relation like $S_T \ln S_T = u'$. We have not specified the ratio l_n/l_{n+1}. Further large-scale simulations of turbulence, at constant energy, should allow determination of a multi-level eddy structure. We wonder if these eddy levels will exhibit the universality seen in other dynamical systems.

Comments

We can relate the proposed eddy structure to the coherent structures that occur in mixing layers, both two and three dimensional (Ashurst, 1979 and Ashurst & Meiburg, 1988). These moving structures produce a smooth time-average Eulerian velocity which does not resemble the Lagrangian flow pattern around any structure. The important point is that flame motion occurs in the Lagrangian frame. The difficulty is that the usual experimental determination is an Eulerian measure. Grasping how different these two viewpoints can be, when molecular mixing determines the phenomena being considered, requires unsteady information, either from simulations or from global diagnostics, such as laser sheet images.

In turbulent combustion the overall relation of S_T versus u' could depend on how the turbulence changes as the magnitude of u' is increased: do more levels of eddys appear in a smooth fashion or as discrete jumps? Is the behavior universal in the Feigenbaum sense, that is can the amplitudes and levels be predicted? There has been abundant work on chaos in systems with linear diffusion, here we have nonlinear diffusion, the G equation, in simple structured flows such as the eddy-quiet-zone pattern – does this lead to similar chaotic behavior?

Most Fourier-based concepts of turbulent flow ignore the phase angle information. She *et al.* (1990) give a graphic illustration of the difference in flow pattern between two systems which have the same energy sprectrum, but one flow is a Navier-Stokes solution

and the other has random phase – the intense vorticity structure, the eddys, do not look at all alike. Flame propagation, we suspect, is very dependent on the actual flow pattern rather than just the energy spectrum.

The iteration equation given by this simple eddy model may allow direct numerical simulations to be iterated to volumes of engineering interest. There appears to be an analogy with progress in molecular dynamics: computer simulation of the hard-sphere fluid led to a polynominal description of pressure versus density. From this base equation of state it is possible to analytically include the effects of an attractive power and the resulting desired equation of state. Similar success was obtained for momentum and thermal transport: computer simulation of soft-sphere viscosity and thermal conductivity gives the density dependence which can be added to the temperature dependence given by kinetic theory (Ashurst & Hoover, 1975).

Therefore, simulation of flame propagation within Navier-Stokes turbulence structure may yield the nonlinear relation of flame speed and turbulence intensity. Addition of larger eddys, outside the range of computer memory, may be accounted for by specification of the coefficients a_n, b_n (assumed invariant in the current work) and u'_n, where n is the eddy level. Which scaling laws and/or fluid mechanical physics should guide the selection of these coefficients? One proposal is the Gurvich-Yaglom model of turbulent energy dissipation (Kolmogorov's third hypothesis), which relates the mean and variance of dissipation to observational volume size, and so the coefficients obtained in a direct simulation could be used to estimate the dissipation statistics at larger sizes (Kerstein & Ashurst, 1984). A second proposal would exploit the observed strain rate behavior dependence upon strain magnitude: the shape of the strain rate tensor tends to a triangular symmetric probability distribution for the intermediate strain rate β at low strain (Ashurst *et al.*, 1987). Hence, the most probable shape is that of plane strain for the larger length scales. So, small-scale direct simulations of flame propagation, combined with an iteration procedure which incorporates the fluid structure, may determine the turbulent flame speed for an arbitrarily large volume of turbulence (arbitrarily large in the galactic sense, *cf.* Gibson, 1991).

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Captions

Figure 1. Computed Navier-Stokes turbulence reveals the tube-like structure of intense vorticity (dark lines) compared to the lack of structure for moderate levels of vorticity (grey lines). The grid size is 96^3 with $Re_\lambda \sim 77$, figure from She, Jackson & Orszag (1991).

Figure 2. Vortical eddy effect upon premixed flame shape. The character of a viscous vortex is a maximum swirl velocity (U_m located at R_m) which creates a flame tip in the advancing flame.

Figure 3. Front propagation through two-dimensional eddys is shown by ten contours of G (any constant G surface represents a flame in this passive model). Propagation is from right to left and each eddy is confined within a radius of 1/2 with the maximum swirl speed at a radius of 1/6. Notice the flame tip formation at the radial location of maximum swirl. This eddy shape resembles a viscous vortex and is described in Kerstein & Ashurst (1992). The swirl rates are $u' = 0.6 S_L$ in a) and 2.0 in b) with grid sizes of 64 by 128, and 128 by 256, respectively. Note the incipient unburned pockets in the eddy core at large u'.

Figure 4. Front advancement rate versus eddy swirl speed u'/S_L for the two-dimensional round-eddy configuration shown in Fig. 3. The numerical results are $S_T/S_L =< |\nabla G| >$ (filled circles) and the model approximation, Eq. (4), is shown by the solid line.

Figure 5. Calculated turbulent flame speed $S_T/S_L =< |\nabla G| >$ in three-dimensional Navier-Stokes turbulence (filled circles) also compares well with the swirling model approximation, Eq. (4), shown by solid line.

Figure 6. The calculated Navier-Stokes turbulent flame speed results given in Figure 5 appear to have a square-root behavior (filled circles: 32^3 grid, triangles: 64^3 grid). The straight line is determined from the slope given by the first and fifth calculated points using the 32^3 grid. This slope ($= 1.32$) combined with the flame speed value at $u' = S_L$ gives a, b values of 0.9021 and 0.1585. The dashed line from $S_T = S_L$ connects the first point with a 4/3 power as found in a random eddy flow at $u' << S_L$ (Kerstein & Ashurst, 1992).

Figure 7. Functional iteration with Eq. (10) can approximate the relation $S_T \ln S_T = u'$, the latter is observed in liquid flame experiments. S_{f_1} is the Navier-Stokes results given in Figure 6 and S_{f_2}, S_{f_3} and S_{f_4} are obtained by changing u'_n, but not the values of a, b.